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The unifying superalgebra $OSp(1|32)$

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Abstract

We show how $OSp(1|32)$ gives a unifying framework to describe $d = 10$ type II string theories, $d = 11$ M-theory and $d = 12$ F-theory. The theories are related by different identifications of their symmetry operators as generators of $OSp(1|32)$. T- and S-dualities are recognized as redefinitions of generators. Some (s,t) signatures of spacetime allow reality conditions on the generators. All those that allow a real structure are related again by redefinitions within the algebra, due to the fact that the algebra $OSp(1|32)$ has only one real realization. The redefinitions include space/space, time/time and space/time dualities. A further distinction between the theories is made by the identification of the translation generator. This distinguishes various versions of type II string theories, in particular the so-called $*$-theories, characterized by the fact that the P_0 generator is not the (unique) positive-definite energy operator in the algebra.

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THE UNIFYING SUPERALGEBRA $OSP(1|32)$

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1 Introduction

The group $OSp(1|32)$ was already mentioned in the first papers on $d = 11$ supergravity [1]. This algebra and its extension $OSp(1|64)$ appeared as anti-de Sitter (adS) and superconformal algebras in $d = 10$ and $d = 11$ Minkowski theories [2] long ago, and got new attention related to the M-theory algebra [3]. The adS or conformal algebras got new attention in a recent paper on the superconformal aspects of $d = 11$ theories [4] and in two-time theories [5]. In these two cases, the $OSp(1|64)$ conformal group appeared. In the physical theories that we consider, we need the subgroup of $OSp(1|64)$ that is a contraction of $OSp(1|32)$ in a way that will be clarified below.

Our initial motivation to study the role of the $OSp(1|32)$ algebra was related to Euclidean theories. When one considers the D-instanton [6], one often considers the bosonic theory, ignoring its possible embedding in the supersymmetric theory. In particular, one makes use of the IIB theory in Euclidean space, while the latter can not be formulated as a supersymmetric theory with real fields, as we will show below. Remark that the connection between these Euclidean theories and the Minkowski string theories involve a duality between theories of different spacetime signature [7].

A second question that was posed when we started this research, was related to the observation that in many super-Euclidean theories one makes use of complexification of the fields and in other cases one does not [8]. We would like to know when it is necessary to do so, and when it can be avoided.

Apart from the possibility of no time directions, one is also interested in theories with more time directions [9, 5, 10]. Therefore, it looked natural to extend our investigation to an arbitrary spacetime signature.

This leads to a web of dualities between theories in $d = 10$, 11 and 12 of different spacetime signature, similar to what has been found in [9]. We obtain these dualities from an algebraic approach, which puts the contraction of $OSp(1|32)$ as a unifying principle. The different theories are then just many faces of the same underlying symmetry group. This seminar summarizes the results obtained in [11]. In [12] we clarify the relation between the super-Poincaré algebra that we consider here, and the full $OSp(1|32)$ as super-adS algebra or $OSp(1|64)$ as superconformal algebra.

Our results are divided in 3 main parts. In section 2 we consider the complex algebra and its realizations in the different dimensions, in section 3 we consider the real algebra and its realizations in different spacetime signatures, and in section 4 we identify the translation operator, distinguishing between different Lagrangian theories for the same spacetime signature. Throughout the work we indicate the dualities connecting all the theories. Finally, a short summary is given in section 5.

2 Complex symmetry algebras

$OSp(1|32)$ is the algebra of 32 fermionic charges with all possible bosonic generators in their anticommutator. We recapitulate its definition, with the bosonic generators defining $Sp(32)$. Then we will see how the contraction, explained in more detail in [12], underlies the F-theory of 12 dimensions, the M-theory of 11 dimensions, and the IIA and IIB string theories in 10 dimensions. They are obtained by identifying appropriate subgroups of $Sp(32)$ as the Lorentz rotations. Note that in the case of the extended algebras mentioned in [12], this $Sp(32)$ is the automorphism algebra of the supersymmetries, in the semi-direct product with $OSp(1|32)$. In any case, the supersymmetries should be in a spinor representation of the Lorentz group. Dimensional reduction and T-dualities are then obtained as mappings between generators of $OSp(1|32)$.

The algebra $OSp(1|32)$ is given by

$$\begin{aligned}
&\{Q_A, Q_B\} = Z_{AB}\,, \qquad [Z_{AB}, Q_C] = Q_{(A}\Omega_{B)C}\,,\\
&[Z_{AB}, Z_{CD}] = \Omega_{A(C}Z_{D)B} + \Omega_{B(C}Z_{D)A}\,,
\end{aligned} \tag{1}$$

where A runs over 32 values, Z_{AB} is symmetric and has thus $\frac{1}{2}.32.33 = 528$ components. Ω_{AB} is an antisymmetric invertible metric, and as such, the last commutator defines $Sp(32)$.

To recognize this algebra as a symmetry algebra in d dimensions, one has to embed $SO(d)$ in $Sp(32)$, in such a way that the spinor representation of $SO(d)$ fits in the 32. This makes already clear that $d = 12$ is the highest possible dimension. To make that identification, we have to select chiral spinors $\hat{Q}$ of 12 dimensions. These are defined using the chiral projection $\hat{\mathcal{P}}^+$:

$$\hat{\mathcal{P}}^+\hat{Q} = \hat{Q}\,, \qquad \hat{\mathcal{P}}^+ = \tfrac{1}{2}(1 + \hat{\Gamma}_*)\,, \qquad \hat{\Gamma}_* = \Gamma_1\Gamma_2\ldots\Gamma_{12}\,. \tag{2}$$

Remark that we use the notation Γ_* (the hat specifies the 12-dimensional context) in any even dimension to denote the product of all the gamma matrices, similar to γ_5 in 4 dimensions. Then the algebra (1) is realized by identifying Ω_{AB} with $\mathcal{C}_{\alpha\beta}$, the (antisymmetric) charge conjugation matrix of $d = 12$. Splitting the matrix Z_{AB} in its irreducible representations, the anticommutator of the supersymmetries looks like

$$\begin{array}{rcccc}
\left\{\hat{Q}, \hat{Q}\right\} & = & \frac{1}{2}\hat{\mathcal{P}}^+\hat{\Gamma}^{\hat{M}\hat{N}}\hat{Z}_{\hat{M}\hat{N}} & + & \frac{1}{6!}\hat{\mathcal{P}}^+\hat{\Gamma}^{\hat{M}_1\cdots\hat{M}_6}\hat{Z}^+_{\hat{M}_1\cdots\hat{M}_6}\,,\\
\frac{1}{2}.32.33 & = & \frac{1}{2}.12.11 & + & \frac{1}{2}\ \frac{12.11.10.9.8.7}{1.2.3.4.5.6}\,,
\end{array} \tag{3}$$

where the last line shows that indeed all 528 generators are present, and the right-hand side thus contains everything which is consistent from the symmetry property of an anticommutator.

In 11 dimensions, the bosonic generators split as $528 = 11 + 55 + 462$, following the anticommutator

$$\{Q_\alpha,\, Q_\beta\} = \Gamma^\mu_{\alpha\beta}P_\mu + 2\Gamma^{\mu\nu}_{\alpha\beta}Z_{\mu\nu} + \frac{1}{5!}\Gamma^{\mu\nu\rho\sigma\tau}_{\alpha\beta}Z^5_{\mu\nu\rho\sigma\tau}\,. \tag{4}$$

In 10 dimensions one can again define chiral spinors, which are 16-dimensional, and consider either 2 generators of opposite chirality (IIA) or of the same chirality (IIB). In the first case, the anticommutators are

$$\begin{aligned}
\left\{Q^\pm, Q^\pm\right\} &= \mathcal{P}^\pm\Gamma^M Z^\pm_M + \tfrac{1}{5!}\mathcal{P}^\pm\Gamma^{M_1\cdots M_5}Z^\pm_{M_1\cdots M_5}\,,\\
\left\{Q^\pm, Q^\mp\right\} &= \pm\mathcal{P}^\pm Z + \tfrac{1}{2}\mathcal{P}^\pm\Gamma^{MN}Z_{MN} \pm \tfrac{1}{4!}\mathcal{P}^\pm\Gamma^{M_1\cdots M_4}Z_{M_1\cdots M_4}\,.
\end{aligned} \tag{5}$$

The 528 generators are thus split as $2 \times (10+126)$ in the anticommutators between generators of the same chirality and 1+45+210 in the anticommutator between generators of opposite chirality.

For the IIB case, we have a doublet of fermionic generators Q^i, $(i=1,2)$, of the same chirality, and the anticommutators are

$$\begin{aligned} \left\{Q^i, Q^j\right\} &= \mathcal{P}^+\Gamma^M Y_M^{ij} + \tfrac{1}{3!}\mathcal{P}^+\Gamma^{MNP}\varepsilon^{ij} Y_{MNP} + \mathcal{P}^+\tfrac{1}{5!}\Gamma^{M_1\cdots M_5} Y^{+\,ij}_{M_1\cdots M_5}\,, \\ Y_M^{ij} &= \delta^{ij} Y_M^{(0)} + \tau_1^{ij} Y_M^{(1)} + \tau_3^{ij} Y_M^{(3)}\,, \end{aligned} \tag{6}$$

where in the second line we have split the symmetric matrix Y^{ij} in three components, as we can also do for the 5-index generators. The decomposition is here $528 = (3\times 10) + 120 + (3 \times 126)$.

It is clear that all these algebras are related. The dimensional reductions relate the generators as follows. The chiral generator $\hat{Q}$ of 12 dimensions, splits in 10 dimensions in a chiral and an antichiral generator, as follows from the relation $\hat{\Gamma}_* = \Gamma_* \otimes \sigma_3$ for a convenient realization of gamma matrices, where Γ_* is the product of 10 gamma matrices of dimension 32×32 in 10 dimensions (for the realization that we use in any dimension see [13]). The two chiral generators are the $Q^\pm$ in (5), and adding them gives the 32-component generator $Q = Q^+ + Q^-$ used for $d=11$. The T-dual theories are identified by taking

$$Q^+ = Q^1\,, \qquad Q^- = \Gamma^s Q^2\,, \tag{7}$$

where Γ^s is a gamma matrix in an arbitrary (spacelike or timelike) direction. On the other hand, S-duality is the mapping

$$Q^i \xrightarrow{S} \left(e^{\mathrm{i}\frac{1}{4}\pi\tau_2}\right)^i{}_j Q^j\,. \tag{8}$$

Thus all the dimensional reductions and dualities are written as mappings between the generators of $OSp(1|32)$. We mentioned here only the fermionic generators explicitly, as the rules for the bosonic generators follow from identifying the anticommutator relations before and after the map. E.g. when going from 12 to 11 dimensions, this leads to the identifications

$$\tilde{Z}_{\tilde{M}} = \mathrm{i}\hat{Z}_{\tilde{M}\,12}\,, \qquad \tilde{Z}_{\tilde{M}\tilde{N}} = \hat{Z}_{\tilde{M}\tilde{N}}\,, \qquad \tilde{Z}_{\tilde{M}_1\cdots\tilde{M}_5} = 2\mathrm{i}\hat{Z}_{\tilde{M}_1\cdots\tilde{M}_5\,12}\,. \tag{9}$$

Note that the appearance of factors i is irrelevant here, as we can make redefinitions of generators involving i at random. For the real algebras, to be discussed below, this should be possible consistently with the reality conditions, as we checked in [11].

3 Real symmetry algebras

The important fact for the real forms is the uniqueness of the real form of the superalgebra $OSp(1|32)$. Therefore the equivalences of all the symmetry algebras of section 2 are valid also for the real form, when it exists. The real form exists only for specific spacetime signatures. The dimensional reduction and T-duality acts now between theories of specific signatures. We have to distinguish then space/space, time/time and space/time dualities.

Considering the table of real forms of the basic Lie superalgebras (see e.g. table 5 of [13] for a convenient presentation), we see that nearly all superalgebras have different real forms, even the exceptional superalgebras. But the algebras $OSp(1|n)$ have only one real form, with $Sp(n,\mathbb{R})$ as bosonic subalgebra.

To realize this real algebra in d dimensions, we have to consider when we can impose consistent reality conditions on the fermionic generators. This is sufficient to be able to classify all the realizations of the unique real superalgebra $OSp(1|32)$. Table 2 of [13] gives the summary of the results that we need. We need 32 real supercharges. The table shows that $d = 12$ with $(space, time)$ signature $(10,2)$ is the highest possible dimension. In general the results are invariant under $(s,t) \simeq (s-4, t+4)$, thus $(6,6)$ is also possible. The interchange of s and t is irrelevant, and corresponds merely to a change of notations of mostly $+$ to mostly $-$ metrics. Therefore we do not mention the $(2,10)$ signature. To make the projections to real spinors one uses three types of projections, Weyl, Majorana or symplectic Majorana:

$$\begin{array}{rcl} W & : & Q = \Gamma_* Q \\ M & : & Q^* = \alpha B Q\,, \quad B \equiv -\mathcal{C}\Gamma_1 \dots \Gamma_t\,, \\ & & BB^* = 1\,, \quad \alpha\alpha^* = 1\,, \\ SM & : & BB^* = -1\,, \quad \alpha\alpha^* = -1\,, \qquad \alpha \text{ antisymmetric matrix.} \end{array} \tag{10}$$

The first one is the Weyl projection to chiral spinors that we already encountered in (2). The other two are reality conditions. B is a definite matrix in spinor space, which is defined here using the charge conjugation matrix $\mathcal{C}$, and all the timelike Γ-matrices. On the other hand, α is a scalar in the spinor space, but may be a matrix acting on the different generators Q, e.g. between Q^+ and Q^- in type IIA, or between Q^1 and Q^2 in type IIB. In the cases where the table indicates 'M', $BB^* = 1$ and a consistent reality condition can be obtained with $\alpha\alpha^* = 1$. Therefore in these cases, a single spinor can suffice, and these are the so-called Majorana spinors. In other cases $BB^* = -1$, consistency (taking the $*$ of a $*$ is an identity) also $\alpha\alpha^* = -1$. Therefore in these cases we need a matrix α of even dimension. We thus have a doubling of the generators, and this is the so-called symplectic Majorana condition. This leads to the possibilities for 32-component real spinors, listed in table 1.

12			(10,2)						(6,6)	
64			MW						MW	
11		(10,1)		(9,2)				(6,5)	=	(5,6)
32		M		M				M		M
10	(10,0)		(9,1)		(8,2)	(7,3)	(6,4)		(5,5)	
32	SM		MW		M	SMW	SM		MW	
	A		A/B		A	B	A		A/B	

Table 1: *The possible spacetime signatures for 32 real spinor generators. The first column indicates the number of complex generators that are present before any projection. The last row indicates, for each signature, whether in $d = 10$ a real form for type IIA (A), type IIB (B) or both (A/B) exists.*

One can then consider the dimensional reductions and T-dualities discussed in section 2, but now we have to be careful with the signatures. When performing the T-duality as in (7), one has to distinguish whether Γ^s is a timelike or a spacelike gamma matrix. This s-direction can even be timelike for the IIA algebra and spacelike for the IIB algebra or vice–versa. These are the time/space or space/time T-dualities, changing the signature. This leads to the both-sided arrows in figure 1. In [11] it was shown that all these identifications of algebras require mappings between the α factors in (10) for the different realizations. At the end, these are all redefinitions of the generators. The reality conditions on the fermionic generators lead to reality conditions on the bosonic generators in order that the anticommutation relations are consistent. The dimensional reduction and T-dualities give mappings that relate real bosonic generators in one algebra with real bosonic generators in the other algebra. This is highly nontrivial, but it is bound to work out, due to the uniqueness of the real form of $OSp(1|32)$.

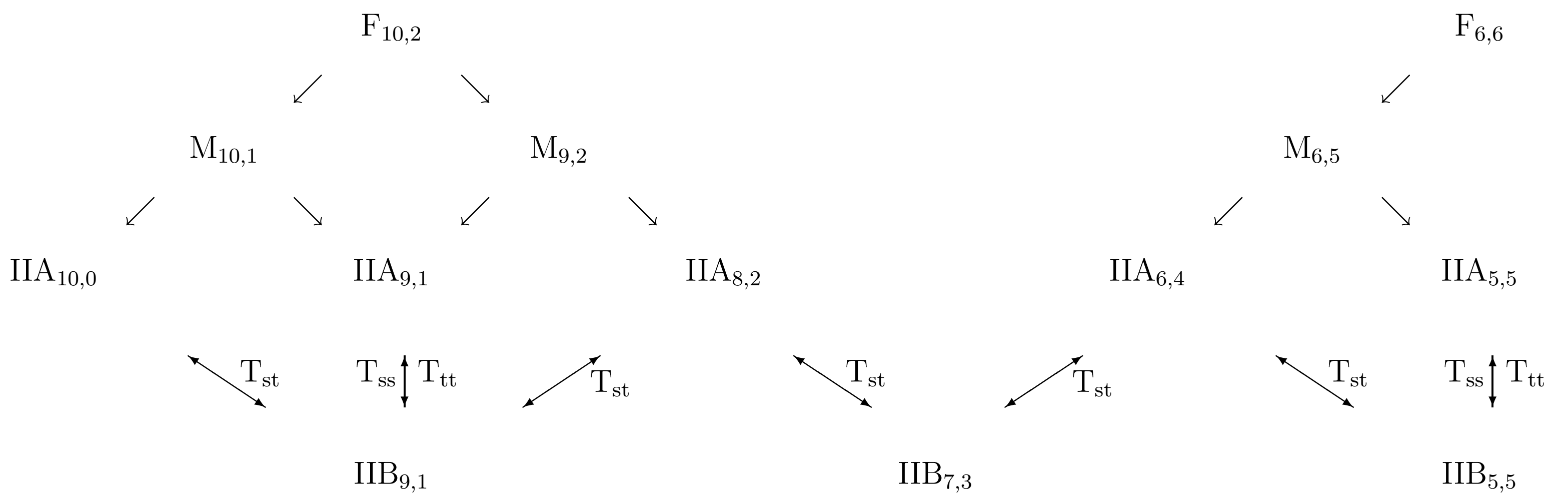

Figure 1: The dimensional reductions and T-dualities between the real algebras in $d = 12$, 11 and 10 of different signatures. The diagonal one-sided arrows indicate dimensional reductions. The both-sided arrows refer to the space/space (T_{ss}), time/time (T_{tt}) and space/time (T_{st}) dualities discussed in the text.

4 Translations and the energy operator

In the third step we identify one of the vector generators as 'translations'. This identification is essential for a spacetime interpretation of the theory. The different possibilities for this identification distinguish e.g. IIA from IIA* theories. We will then remark that T-duality gives a mapping between different types of generators. It can mix e.g. translations with 'central charges'. Finally, we will see that there is a unique positive energy operator in the algebra. However, that generator is not always the timelike component of the translations. For instance, in IIA theories, the positive operator is P_0, but in IIA* theories it is another one, and thus P_0 is not positive in that case.

So far, all bosonic generators were treated on equal footing. To make the connection between algebras and a spacetime theory, we want to know which generator performs 'translations' in spacetime. Seen in another way, spacetime is the manifold defined from a base point by the action of this 'translation' generator. This is thus similar to the coset space idea. To generate a spacetime of the appropriate dimension, the translation operator should be a vector operator in the theory. This is nearly the only requirement, apart from a non-degeneracy condition. Indeed, in order that the supersymmetries perform their usual role, they should square to the translations. Thus the matrix that appears in the anticommutator between all the supersymmetries, defining how they square to translations, should be non-degenerate.

For $d = 12$, with the algebra (3), there is no vector operator. Thus there is no candidate for translations, implying that F-theory has no straightforward spacetime interpretation. On the other hand, for $d = 11$, with the algebra (4), there is one vector operator, and this one should thus be called the translation generator.

In 10 dimensions it becomes more interesting. Consider first the IIA algebra (5). There are 2 vector operators Z^+_M and Z^-_M. Both separately are not convenient, because then one half of the supersymmetries would not square to translations. But we can take linear combinations. For the signature $(9,1)$ there are, up to redefinitions, two choices consistent with the reality conditions

$$\begin{aligned} (9,1)\ :\ \text{IIA} \quad &:\ P_M \equiv Z^+_M + Z^-_M\,,\\ \text{IIA}^* \quad &:\ P_M \equiv Z^+_M - Z^-_M\,. \end{aligned} \tag{11}$$

We label these choices as IIA and IIA* in accordance with [9]. For signatures $(10,2)$ or $(8,2)$ there are the possibilities

$$(10,0) \text{ or } (8,2)\ :\ \text{IIA} \quad :\ P_M \equiv \mathrm{i}(Z^+_M + Z^-_M)\,,$$

$$\mathrm{IIA}' \ : \ P_M \equiv Z^+_M - Z^-_M \,. \tag{12}$$

However, now these two choices can be related by a redefinition $Q^{\pm} \to e^{\pm i\pi/2} Q^{\pm}$. Such a redefinition is, similar to (8) and therefore we recognize it as an S-duality.

The operators that are not translations, remain as ‘central charges’ in the theory. Therefore we see that the generator that is a translation in one theory, appears as a central charge in the other theory, as we announced in the beginning of this section.

For the IIB algebra (6), there are three candidates for translations. For signature (9,1) they are all consistent with the reality condition. We thus distinguish

$$\begin{array}{lll} (9,1)\ : \qquad & \mathrm{IIB} & : \ P_M = Y^{(0)}_M \,, \\ & \mathrm{IIB}^* & : \ P_M = Y^{(3)}_M \,, \\ & \mathrm{IIB}' & : \ P_M = Y^{(1)}_M \,. \end{array} \tag{13}$$

Considering the possible redefinitions, we come back to (8). This S-duality leaves the IIA translation generator invariant, and relates the translation of IIB^* with that of IIB'. On the other hand, for the signature (7,3) there are three S-dual versions.

We present the results schematically in figure 2. It is a detail of a part of figure 1. The specification of the translation generator distinguishes different IIA and IIB theories. The T-dualities connect specific versions. S-dual theories are mapped by T-dualities to S-dual theories.

Finally, we can identify one bosonic operator in $OSp(1|32)$ that is positive. We can identify this one from the anticommutation relation

$$\left\{Q^i, Q^{j\,*}\right\} = \left\{Q^i, Q^j\right\} B^T \geq 0 \,, \tag{14}$$

using the Majorana condition[1] in (10). We denote

$$\left\{Q^i, Q^j\right\} = \mathcal{Z}^{ij} \mathcal{C}^{-1} \tag{15}$$

to represent all the anticommutation relations, where $\mathcal{Z}^{ij}$ is a matrix in spinor space as well as in the i, j indices. For convenience we write here the charge conjugation matrix explicit. Using the expression of B in (10), this implies

$$\mathcal{Z}^{ij} \Gamma^t \cdots \Gamma^1 \geq 0 \,. \tag{16}$$

[1] We take here $\alpha = 1$ and, to argue for the positivity, we use the convention that complex conjugation does not change the order of the operators. See [11] for the arguments independent of these conventions

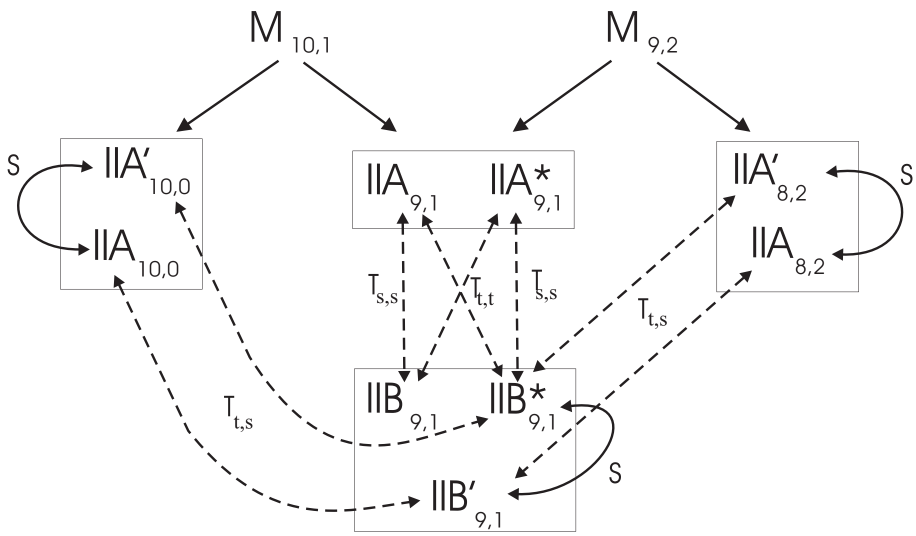

Figure 2: *Dimensional reduction, T and S-dualities after specification of the translation operator.*

Therefore, the trace of that operator has positive eigenvalues. When we split Z as usual in different irreducible representations for the spacetime Lorentz group, then, in order to absorb the gamma matrices, the relevant part of Z has as many spacetime indices as there are time directions. All its directions should be timelike. Thus for Minkowski spaces it is the timelike component of a vector operator, while for Euclidean theories this positive operator is a scalar 'central charge'. For Minkowski theories we can thus wonder whether the positive energy operator is the timelike component of the operator that we selected as 'translations'. If this is the case then the usual Hamiltonian will be positive. When the positive energy is the timelike component of another vector operator, then the Hamiltonian built from P_0 is not positive definite. This is what happens in the IIA*, IIB* and IIB$'$ theories. In these theories, the kinetic energies of some of the p-form gauge fields are negative definite. As an example consider the vector operators in the IIB-like theories, as in (6). With our convention that $\alpha = 1$, the trace in (16) selects the $M = 0$ component of $Y_M^{(0)}$ as the positive definite energy. Thus it is indeed the IIB theory where the energy is the timelike part of translations, and not for the other versions. The algebraic approach thus gives an understanding of the positivity in type IIA and IIB versus lack of positivity in the other theories.

5 Conclusions

The algebras of F-theory, M-theory, type IIA and IIB, ... are different faces of the same superalgebra $OSp(1|32)$. The uniqueness of the real form of that algebra implies that all these manifestations can be related by mappings between the generators of the algebra. That holds especially for the dimensional reductions, T- and S-dualities that relate these theories. Different spacetime signatures are easily incorporated. However, for certain spacetime signatures, some theories may exist only in complex form. That answers the questions about why we need sometimes a complexification procedure to obtain an Euclidean theory. In particular, the IIB theory has no real form in (10,0). Therefore, in order to discuss the D-instanton in IIB, we have to give up the concept of a theory with real fields and action.

We have understood the *-theories as being distinguished from the usual IIA and IIB by a different identification of the translation generator. They are related to the ordinary theories by a 'duality' interchanging translations with central charges. Thus in these dualities the concept of spacetime is very intriguing. It should be interchanged with a sort of harmonic space where coordinates are associated also with other (vector) central charges. The unique positive energy operator is the timelike component of the translations in the ordinary type IIA and IIB theories, but in other versions (*-theories or theories with a different signature), it is not the P_0 operator that is positive, but rather a component of a central charge operator.

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[1] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. **B76** (1978) 409.

[2] J. W. van Holten and A. Van Proeyen, J. Phys. A **A15** (1982) 3763.

[3] P. K. Townsend, in 'Strings, branes and dualities', Cargèse 1997, ed. L. Baulieu et al., Kluwer Academic Publ., 1999, p.141 [hep-th/9712004]; M. Günaydin, Nucl. Phys. **B528** (1998) 432 [hep-th/9803138].

[4] P. West, JHEP **0008** (2000) 007 [hep-th/0005270].

[5] I. Bars, C. Deliduman and D. Minic, Phys. Rev. **D59** (1999) 125004 [hep-th/9812161]; Phys. Lett. **B457** (1999) 275 [hep-th/9904063];
I. Bars, see these proceedings, [hep-th/0008164].

[6] J. Polchinski, Phys. Rev. **D50** (1994) 6041 [hep-th/9407031];
M. B. Green, Phys. Lett. **B354** (1995) 271 [hep-th/9504108];
G. W. Gibbons, M. B. Green and M. J. Perry, Phys. Lett. **B370** (1996) 37 [hep-th/9511080].

[7] G. Moore, [hep-th/9305139];
G. Moore, in *'Supersymmetry and unification of fundamental interaction' (Susy 93)*, ed. P. Nath. World Scientific, 1993 [hep-th/9308052];
E. Bergshoeff, C. Hull and T. Ortín, Nucl. Phys. **B451** (1995) 547 [hep-th/9504081];
K. Behrndt and E. Bergshoeff, Class. Quant. Grav. **15** (1998) 1801 [hep-th/9803090];
E. Cremmer, I. V. Lavrinenko, H. Lu, C. N. Pope, K. S. Stelle and T. A. Tran, Nucl. Phys. **B534** (1998) 40 [hep-th/9803259];
A. V. Belitsky, S. Vandoren and P. van Nieuwenhuizen, Phys. Lett. **B477** (2000) 335 [hep-th/0001010].

[8] B. Zumino, Phys. Lett. **69B** (1977) 369;
H. Nicolai, Nucl. Phys. **B140** (1978) 294;
P. van Nieuwenhuizen and A. Waldron, Phys. Lett. **B389** (1996) 29 [hep-th/9608174].

[9] C. M. Hull, JHEP **9807** (1998) 021 [hep-th/9806146];
C. M. Hull and R. R. Khuri, Nucl. Phys. **B536** (1998) 219 [hep-th/9808069].

[10] C. N. Pope, A. Sadrzadeh and S. R. Scuro, Class. Quant. Grav. **17** (2000) 623 [hep-th/9905161];
H. Nishino and S. J. Gates, Class. Quant. Grav. **17** (2000) 2139 [hep-th/9908136].

[11] E. Bergshoeff and A. Van Proeyen, Class. Quant. Grav. **17** (2000) 3277 [hep-th/0003261].

[12] E. Bergshoeff and A. Van Proeyen, proceedings of the Gürsey Memorial Conference, to be published in Class. Quantum Grav.

[13] A. Van Proeyen, Physics AUC **9** (part I) (1999) 1 [hep-th/9910030].